Towards the seislet transform

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SUMMARY

I introduce a digital wavelet-like transform tailored specifically for representing seismic data. The transform provides a multiscale orthogonal basis with basis functions aligned along seismic event slopes in the input data. It is defined with the help of the wavelet lifting scheme combined with local plane-wave destruction. The main objective of the new "seislet" transform is an optimal seismic data compression for designing efficient algorithms. Traditional signal processing tasks such as noise removal and trace interpolation are simply defined in the seislet domain and realizable with optimally efficient $O(N)$ algorithms. When applied in the offset direction on common midpoint or common image point gathers, the seislet transform finds an additional application in optimal stacking of seismic records.

INTRODUCTION

Wavelet transforms have found many applications in science and engineering (Mallat, 1999), including geophysics (Foster et al., 1994; Dessing, 1997; Wapenaar et al., 2005). The power of wavelet transforms, in comparison with the classic Fourier transform, lies in their ability to represent non-stationary signals. As a result, wavelets provide an optimally compact basis for non-stationary data decomposition. Having a compact basis is useful both for data compression and for designing efficient numerical algorithms.

Recently, a number of wavelet-like transforms that explore directional characteristics of images, have entered the image analysis literature (Welland, 2003). Among those transforms are bandelets (Pennec and Mallat, 2005), contourlets (Do and Vetterli, 2005), curvelets (Starck et al., 2000), directionlets (Velisavljevic, 2005), etc. Unlike isotropic wavelets, directional-type transforms attempt to design basis functions that are elongated anisotropically along 2-D curves or 3-D surfaces that might be characteristic for an image. Therefore, they achieve better accuracy and better data compression in representing non-stationary images with curved edges. Curvelets seem particularly appropriate for seismic data because they provide provably optimal decomposition of wave-propagation operators (Candès and Demanet, 2004). Application of the curvelet transform to seismic data analysis is an area of active research (Herrmann, 2003; Douma and de Hoop, 2005).

Although wavelet theory originated in seismic data analysis (Morlet, 1981), none of the known wavelet-like transforms were designed specifically for seismic data. Even though some of the transforms are applicable to representing seismic data, their original design was motivated by a completely different kinds of data, such as piecewise-smooth images. In this paper, I investigate the possibility of designing a transform tailored specifically for seismic data. In analogy with previous naming games, I call such a transform *the seislet transform*[1].

The approach taken in this paper follows the general recipe for digital wavelet transform construction known as the *lifting scheme* (Sweldens, 1995). The lifting scheme provides a convenient and efficient construction for digital wavelet transforms of different kinds. The key ingredients of the scheme are a prediction operator and an update operator defined at different digital scales. The goal of the prediction operator is to predict regular parts of the image so that they could be subtracted from the analysis. The goal of the update operator is to carry essential parts of the image to the next analysis scale. Conventional wavelet transforms use prediction and update operators designed for characterizing locally smooth images. In this paper, I show how designing prediction and update suitable for seismic data can improve the effectiveness of the transform in seismic applications. I use prediction along locally dominant event slopes found by the method of plane-wave destruction (Fomel, 2002).

The seislet transform decomposes a seismic image into an orthonormal basis which is analogous to the wavelet basis but aligned along dominant seismic event slopes. Using synthetic and field data examples, I demonstrate the effectiveness of the new transform in characterizing seismic data and in accomplishing traditional signal processing tasks such as noise removal, trace interpolation, and stacking.

[1]Name suggested by Huub Douma (pers. comm.)

TRANSFORM CONSTRUCTION

(a) (b)

Figure 1: Benchmark "Lena" image from image analysis literature (a) and its 2-D digital wavelet transform using bi-orthogonal wavelets (b).

In order to define the new transform, I follow the general recipe for digital wavelet transforms provided by Sweldens and Schröder (1996). In the most general terms, the lifting scheme (Sweldens, 1995) is defined as follows:

1. Organize the input data as a sequence of records.
2. Break the data into even and odd components **e** and **o**.
3. Find a residual difference **r** between the odd component and its prediction from the even component:
$$\mathbf{r} = \mathbf{o} - \mathbf{P}[\mathbf{e}] \; , \tag{1}$$
where **P** is a *prediction* operator.
4. Find a coarse approximation **c** of the data by updating the even component
$$\mathbf{c} = \mathbf{e} + \mathbf{U}[\mathbf{r}] \; , \tag{2}$$
where **U** is an *update* operator.
5. The coarse approximation **c** becomes the new data, and the sequence of steps is repeated at the next scale level.

A digital wavelet transform consists of data approximation at the coarsest level and residuals from all the levels. The key in designing an effective transform is making sure that the prediction operator **P** leaves small residuals while the update operator **U** preserves essential features of the original data while promoting them to the next scale level. For example, one can obtain the $(2,2)$ Cohen-Daubechies-Feauvea bi-orthogonal wavelets (Cohen et al., 1992) by making the prediction operator a linear interpolation between two neighboring samples

$$\mathbf{P}[\mathbf{e}]_k = \mathbf{e}_{k-1} + \mathbf{e}_k \tag{3}$$

and by constructing the update operator to preserve the running average of the signal (Sweldens and Schröder, 1996)

$$\mathbf{U}[\mathbf{r}]_k = (\mathbf{r}_{k-1} + \mathbf{r}_k)/4\,. \tag{4}$$

The digital wavelet transform is an efficient operation. Assuming the prediction and update operation take a constant cost per record, the number of operation at the finest scale is proportional to the total number of records N, the next scale computation takes $O(N/2)$, etc. so that the total number of operations is proportional to $N + N/2 + N/4 + \ldots + 2 = 2(N-1)$, which is smaller than $O(N \log N)$ cost of the Fast Fourier Transform.

The transform is also easily invertible. Reversing the lifting scheme operations provides the inverse transform algorithm, as follows:

1. Start with the coarsest scale data representation **c** and the coarsest scale residual **r**.
2. Reconstruct the even component **e** by reversing the operation in equation 2, as follows:

$$\mathbf{e} = \mathbf{c} - \mathbf{U}[\mathbf{r}]\,, \tag{5}$$

3. Reconstruct the odd component **o** by reversing the operation in equation 1, as follows:

$$\mathbf{o} = \mathbf{r} + \mathbf{P}[\mathbf{e}]\,, \tag{6}$$

4. Combine the odd and even components to generate the data at the previous scale level and repeat the sequence of steps.

Figure 1 shows a classic benchmark image from the image analysis literature and its digital wavelet transform using two-dimensional bi-orthogonal wavelets. Thanks to the general smoothness of the "Lena" image, the residual differences from equation 2 (stored as wavelet coefficients at different scales) have a small dynamic range, which allows for an effective compression of the image.

FROM WAVELETS TO SEISLETS

I adopt the general idea of the lifting scheme to transforming multidimensional seismic data. The key idea of the seislet transform is recognizing that

- seismic data should be organized as a collection of traces or records and not simply as a collection of samples;
- prediction of one seismic trace or record from the other and update of records on the next scale should follow features characteristic for seismic data.

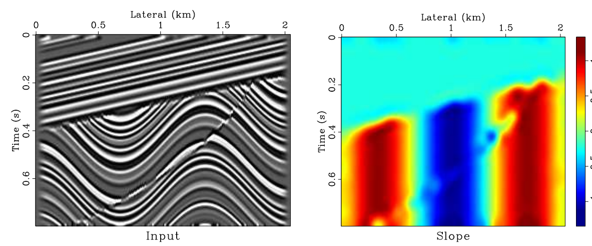
For example, one can view seismic data as collections of traces and predict one trace from the other by following local seismic event slopes. Such a prediction is a key operation in the method of plane-wave destruction (Fomel, 2002). In fact, it is the minimization of prediction error that provides a criterion for estimating local slopes (Claerbout, 1992).

The prediction and update operators for a simple seislet transform are defined by modifying the bi-orthogonal wavelet construction in equations (3-4) as follows:

$$\mathbf{P}[\mathbf{e}]_k = \mathbf{S}_k^{(+)}[\mathbf{e}_{k-1}] + \mathbf{S}_k^{(-)}[\mathbf{e}_k] \tag{7}$$

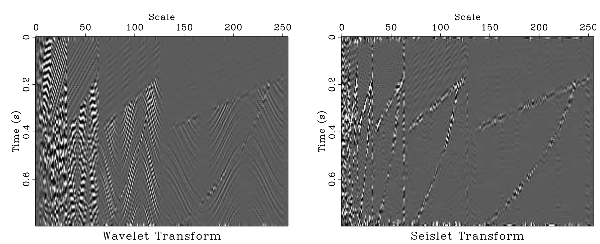
$$\mathbf{U}[\mathbf{r}]_k = \left(\mathbf{S}_k^{(+)}[\mathbf{r}_{k-1}] + \mathbf{S}_k^{(-)}[\mathbf{r}_k]\right)/4\,, \tag{8}$$

where $\mathbf{S}_k^{(+)}$ and $\mathbf{S}_k^{(-)}$ are operators that predict a trace from its left and right neighbors correspondingly by shifting seismic events according to their local slopes. The predictions need to operate at different scales, which, in this case, mean different distances between the traces. Equations (7-8), in combination with the forward and inverse lifting schemes (1-2) and (5-6), provide a complete definition of a useful transform.



(a) (b)

Figure 2: Synthetic seismic image (a) and local slopes estimated by plane-wave destruction (b).



(a) (b)

Figure 3: Wavelet transform (a) and seislet transform (b) of the synthetic image from Figure 2.

Figure 2(a) shows a synthetic seismic image from Claerbout (2006). Estimating local slopes from the image (Figure 2(b)), I applied the seislet transform described above. The transform is shown in Figure 3(b) and should be compared with the corresponding wavelet transform in Figure 3(a). Apart from the fault and unconformity regions, where the image is not predictable by continuing local slopes, the seislet transform coefficients are small which allows for an effective compression. The wavelet transform has small residual coefficients at the fine scale but develops large coefficients at coarser scales.

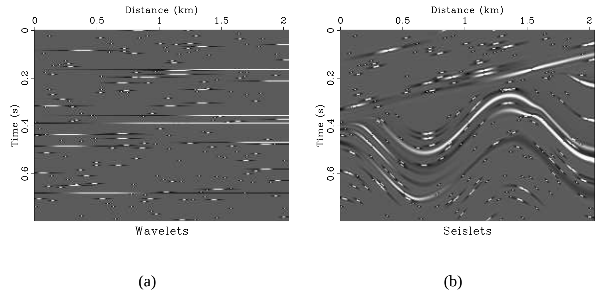


Figure 4: Randomly selected representative basis functions for wavelet transform (a) and seislet transform (b).

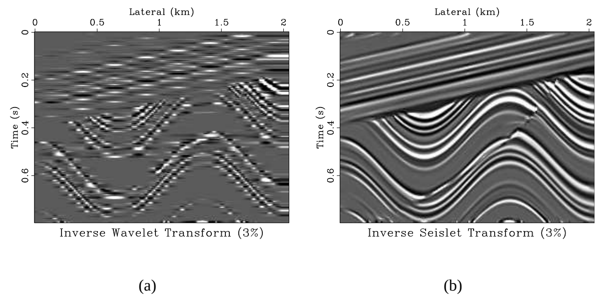


Figure 5: Image reconstruction using only 3% of significant coefficients (a) by inverse wavelet transform (b) by inverse seislet transform.

Effectively, the wavelet transform in this case is equivalent to the seislet transform for the erroneous zero slope. Figure 4 shows example basis functions for the wavelet and seislet transform used in this example. Expectedly, wavelet transform fails to reconstruct the most important features of the original image when using only the most significant coefficients while the seislet transform achieves an excellent reconstruction (Figure 5). I used the method of soft thresholding (Donoho, 1995) for selecting the most significant coefficients.

Figure 6(a) shows a common-midpoint data from a real marine dataset. Plane-wave destruction estimates local slopes shown in Figure 6(b) and enables the seislet transform shown in Figure 7(a). Small dynamic range of the seislet coefficients implies a good compression ratio. Figure 7(b) shows data reconstruction using only 5% of the significant seislet coefficients.

If we choose the significant coefficients at the coarse scale and zero out difference coefficients at the finer scales, the inverse transform will remove incoherent noise from the gather (Figure 7). Thus, denoising is a naturally defined operation in the seislet domain.

If we extend the seislet domain, possibly with random noise, and interpolate the smooth local slope to a finer grid, the inverse seislet transform will accomplishes trace interpolation of the input gather (Figure 9). In this example, the number of traces is increased by four. Thus, trace interpolation also turns out to be a natural operation when viewed from the seislet domain.

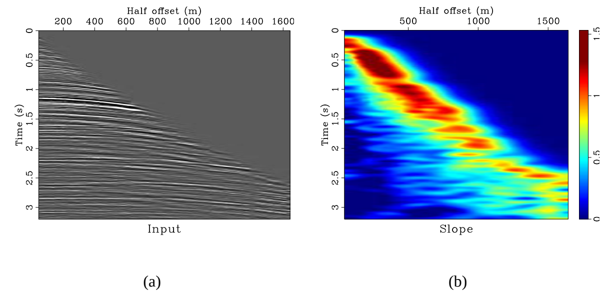


Figure 6: Common-midpoint gather (a) and local slopes estimated by plane-wave destruction (b).

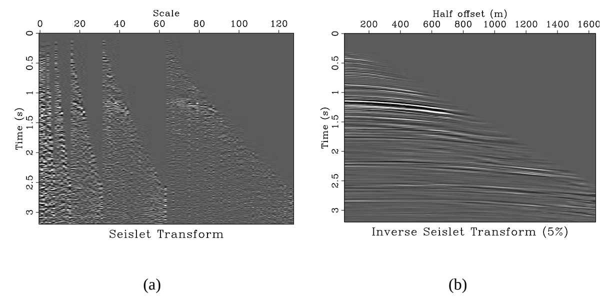


Figure 7: Seislet transform of the input gather (a) and (a) and data reconstruction using only 5% of significant seislet coefficients (b).

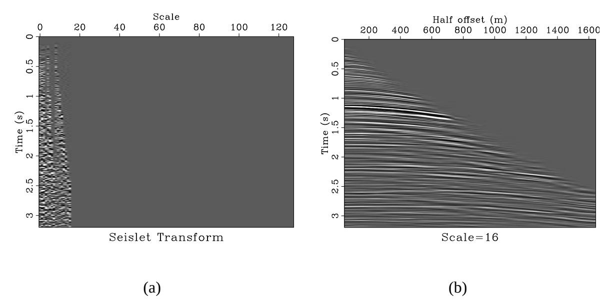


Figure 8: Zeroing seislet difference coefficients at fine scales (a) enables effective denoising of the reconstructed data (b).

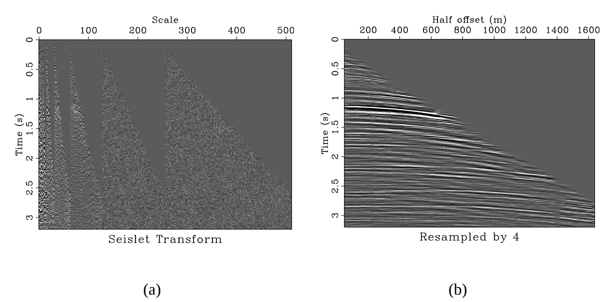


Figure 9: Extending seislet transform with random noise (a) enables trace interpolation in the reconstructed data (b).

SEISLET STACK

The seislet transform defined in the previous section acquires a special meaning when applied in the offset direction on common midpoint or common image point gathers. According to the lifting construction, the zero-order seislet coefficient is nothing more than seismic stack but computed in a recursive manner by successive partial stacking of neighboring traces. As a consequence, seislet stack avoids the problem of "NMO stretch" associated with usual stacking as well as the problem of nonhyperbolic moveouts. All other gather attributes including multiple reflections and amplitude variation with offset appear in the higher order seislet coefficients. Figure 10 shows a comparison between the conventional NMO stack and the seislet stack. The higher resolution of the seislet stack is clearly visible. Figure 11 compares the common-midpoint gather after conventional normal moveout correction and an effective seislet moveout computed by separating contributions from individual traces to the stack.

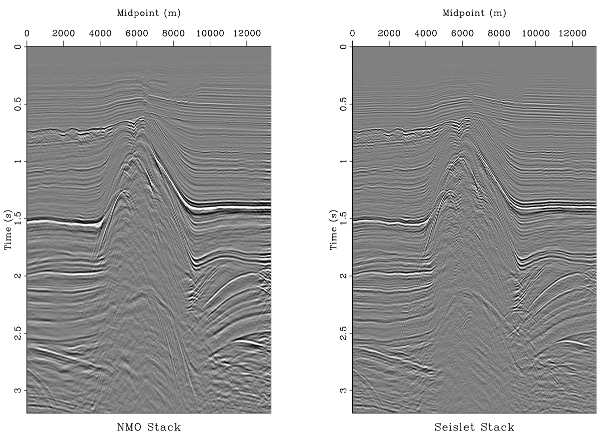
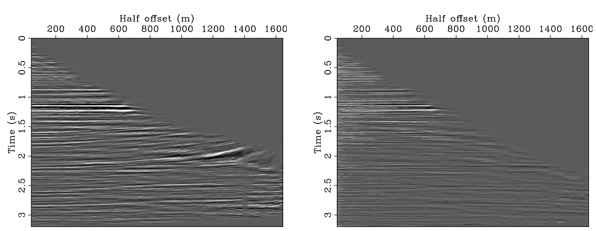


Figure 10: Left: conventional normal-moveout stack. Right: seislet stack.



(a) (b)

Figure 11: Input gather after normal moveout correction (a) and effective seislet moveout (b).

CONCLUSIONS

I have introduced a new digital transform named "seislet transform" because of its ability to characterize and compress seismic data in the manner similar to that of digital wavelets transforms. I defined the seislet transform by combining the wavelet lifting scheme with local plane-wave destruction.

The new transform provides a convenient orthonormal basis with the basis functions spanning different scales analogously to those of the digital wavelet transform but aligned anisotropically along the dominant seismic slopes. Traditional signal analysis operations such as denoising and trace interpolation become simply defined in the seislet domain and allow for optimally efficient algorithms. Seismic stack also has a simple meaning of the zeroth-order seislet coefficient computed in an optimally efficient manner by recursive partial stacking, thus avoiding the usual problems with wavelet stretch and nonhyperbolic moveouts. One can extend the idea of the seislet transform further by changing the definition of prediction and update operators in the lifting scheme.

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